

## Combining the Tangent Portfolio Concept with LDI

Ralph P. Goldsticker III, CFA  
*Managing Director,  
 Senior Investment Strategist,  
 Institutional Relationship  
 Management Group,  
 BNY Mellon  
 Asset Management*

### INTRODUCTION

Many defined benefit pension fund plan sponsors have been shifting their asset allocations towards liability-driven investing (LDI) in an effort to reduce the impact of the plan on their financial statements. Others have been utilizing strategies that lever a high Sharpe ratio portfolio in an effort to improve the risk-adjusted return of their plan assets. Frequently the decisions about how much to hedge and how to construct the “optimal” return-seeking portfolio are made independently. An indication of this fact is that sponsors often refer to their portfolios in terms of hedging assets and growth assets. This article demonstrates how the two approaches can be combined in a way that is designed to reduce the volatility of the plan and achieve maximum benefit from the plan’s “active risk.”<sup>1</sup>

We illustrate the relationships using a series of efficient frontiers.<sup>2</sup> They provide visual illustrations of the trade-off between a portfolio’s expected return and its expected risk, and they demonstrate that most portfolio allocations result in two sources of risk to the plan: the liabilities are less than fully hedged, and the portfolio includes risky (return-seeking or growth) assets.

<sup>1</sup> The term “active risk” refers to the risk of the total plan (assets minus liabilities). For example, consider a plan with \$1 billion of assets and \$1 billion of liabilities. If the assets return 10% and the liabilities grow by 5%, the value of the plan will increase by \$50 million. We view this as a 5% active return (a \$50 million gain on a \$1 billion base). The reason we use the term “active risk” is that we assume that the liabilities can be fully hedged, so that any risk taken relative to the liability-hedging portfolio is the result of an “active” decision. We acknowledge that it is not possible to perfectly hedge the liabilities, so active risk cannot truly be zero. However, normally most of the active risk is the result of the decision not to fully hedge. Consequently, we will not attempt to isolate the relatively small amount of active risk that comes from a less-than-perfect liability hedge.

<sup>2</sup> Efficient frontiers are the set of portfolios that have the maximum expected return for each level of risk. They are generated using an optimizer that requires estimates of expected returns, expected risks, and correlations. We believe the parameters used in this analysis are consistent with long-term equilibrium relationships. They are not intended to be forecasts of future expectations. We believe the risk-return relationships illustrated in the article exist regardless of the specific inputs, while asset weights can be highly sensitive to small changes in inputs. Therefore, we do not show the asset weights for any of the portfolio mixes. For the purposes of this article, all of the calculations assume that the plan is exactly 100% funded. Over- or underfunding would change the estimates of the risk and return statistics, but the article’s conclusions would remain the same.

## ASSET-ONLY EFFICIENT FRONTIERS: FIGURES 1A, 1B AND 1C

The first three charts illustrate the benefit of leveraging the tangent (maximum Sharpe ratio) portfolio. *Figure 1a* shows the traditional “asset only” view of portfolio risk and return. It shows the traditional efficient frontier, with portfolio risk (expected standard deviation of portfolio returns) on the horizontal axis, and portfolio expected return on the vertical axis. The chart also includes points showing the risk and return for several asset classes. Since this chart doesn’t include cash, the minimum-risk portfolio is almost entirely invested in an aggregate bond fund, the asset class with the lowest risk. The traditional approach to asset allocation is to evaluate all of the portfolios on the efficient frontier and select the one that is expected to provide the “best” risk-adjusted return.

Risk-averse investors will pick portfolios at the lower end of the frontier, and investors who are more aggressive will pick portfolios at the riskier end of the curve. The portfolio labeled “Efficient 60/40 mix” is the portfolio on the efficient frontier that has an allocation of 60% equities and 40% fixed income. Of course, there are many other portfolios that have a 60/40 mix, but they lie below the efficient frontier and are dominated by portfolios that lie on the efficient frontier.

*Figure 1b* adds cash to the analysis. Now the minimum-risk portfolio has only 0.5% volatility and has the commensurately low expected return of 3.5%. The tangent point is where a straight line from cash is tangent to the efficient frontier. The tangent point also identifies the portfolio that theoretically has the maximum Sharpe ratio.<sup>3</sup> Note that once we add cash, the efficient portfolios below the point of tangency are a combination of cash and the tangent portfolio (the orange line).

*Figure 1c* shows what happens if we can employ leverage. The efficient frontier becomes a straight line starting at 100% cash, passing through the tangent portfolio, and continuing to the upper right. Portfolios below the tangent portfolio contain a combination of cash and the tangent portfolio. Portfolios beyond the tangent portfolio are levered versions of the tangent portfolio. The farther out they are from the tangent portfolio, the more leverage they require. This chart makes the case for why we believe that leverage can be a useful tool. It allows us to create portfolios that are more efficient—having higher expected returns per unit of risk—than we can by simply moving out the long-only efficient frontier.<sup>4</sup>

The arrows starting at the “Efficient 60/40 mix” portfolio highlight the range of portfolios on the levered efficient frontier that dominate the 60/40 portfolio. The vertical arrow points to the portfolio with the same risk but a higher expected return, and the horizontal line points to the portfolio with the same expected return but lower risk. The expected gain from utilizing leverage will depend on the shapes of the levered and unlevered efficient frontiers. The steeper the levered efficient frontier and the flatter the unlevered, the greater the gain will be from using leverage. Conversely, the flatter the levered and the steeper the unlevered, the smaller the benefit.

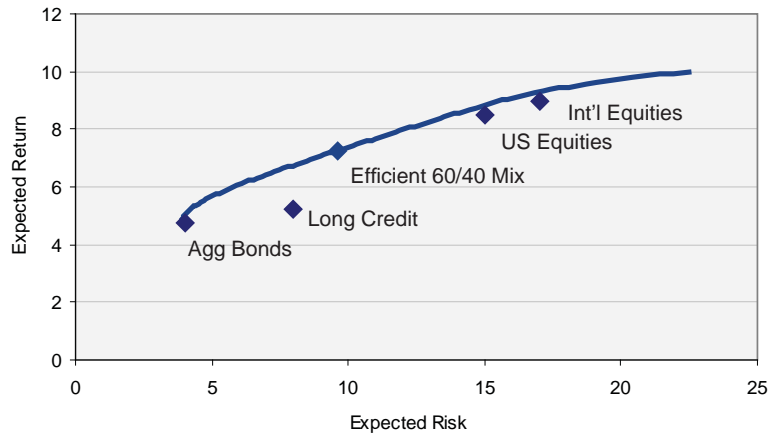
<sup>3</sup> Sharpe ratio = (portfolio expected return – risk-free rate) / portfolio risk.

<sup>4</sup> The tangent portfolio concept is an integral part of the Capital Asset Pricing Model (CAPM). In it, the tangent portfolio is asserted to be the market portfolio, and the levered efficient frontier is called the Capital Market Line.

**Figure 1a: Long-Only Efficient Frontier**

Assets Only Efficient Frontier – No Cash, No Leverage

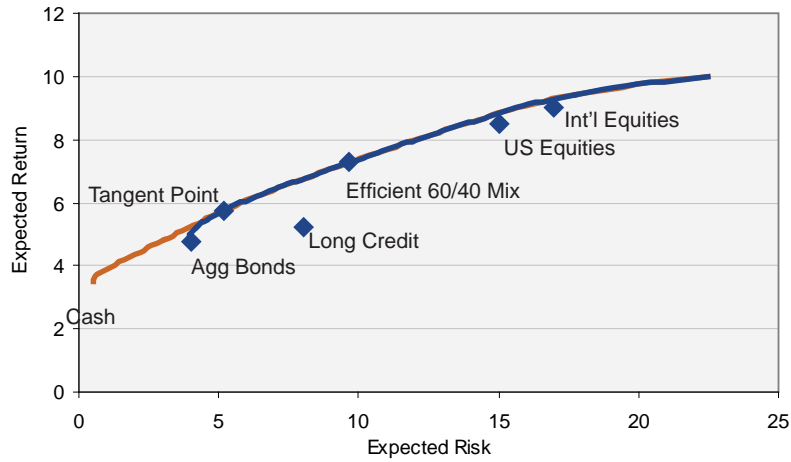
Data Source: Mellon Capital



**Figure 1b: Long-Only Efficient Frontier, Including Cash**

Assets Only Efficient Frontier – Effect of Adding Cash

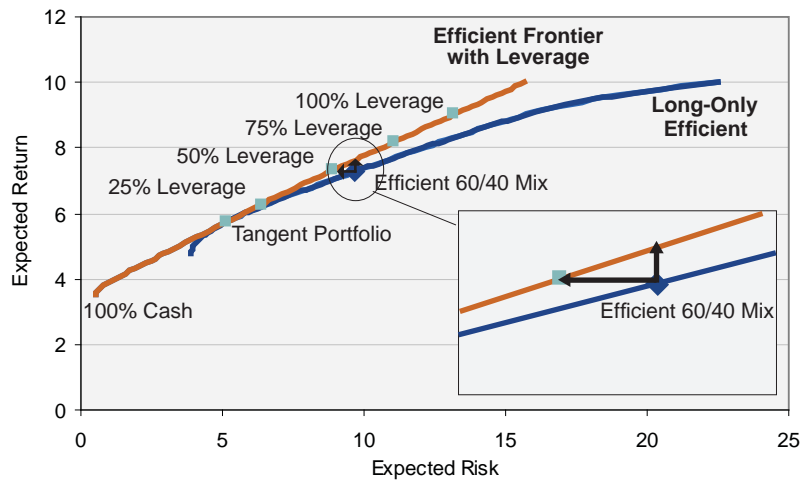
Data Source: Mellon Capital



**Figure 1c: Long-Short Efficient Frontier, Including Leverage**

Assets Only Efficient Frontier – Effect of Allowing Leverage

Data Source: Mellon Capital



## ASSET-LIABILITY EFFICIENT FRONTIERS: FIGURES 2A, 2B AND 2C

So far, we have been operating in an asset-only framework and have been calculating portfolio risk without consideration of the plan's liabilities. *Figure 2a* reframes the efficient frontier into an asset-liability context.<sup>5</sup> The horizontal axis is now the active risk of the total plan,<sup>6</sup> and the vertical axis is the expected return of the plan assets.<sup>7</sup> This chart has three lines. The dark blue line is calculated using the portfolio mixes from the asset-only efficient frontier (*Figure 1a*), but risk is now calculated in terms of the total plan. Using that frame of reference, the risk of the asset-only minimum-risk portfolio is now almost 9%. That is much higher than the 4% estimate of portfolio risk when liabilities were excluded from the calculation.<sup>8</sup>

The green line is the efficient frontier optimized referencing the active risk of the plan. It shows a significant improvement. Its minimum-risk portfolio has a volatility of about 6%. The orange line is the efficient frontier optimized with a custom LDI portfolio.<sup>9</sup> The minimum risk on that efficient frontier is approximately 3.5%. It's not zero because we assume that the correlation between the LDI portfolio and the liabilities is only 0.95. We believe this line illustrates three precepts: one should consider the plan's liabilities when evaluating a portfolio's risk; optimizations that include the liabilities will produce more efficient portfolios than ones that exclude them; and strategies that incorporate custom LDI portfolios will be the most efficient.

As we seek higher expected returns, all three lines converge. That is because as the portfolios contain more equities, the hedging aspects of LDI and long-bond strategies are reduced.

The chart also shows points for the long-only "60/40 efficient mix" and portfolios in which we replace the fixed income component of that portfolio with either long-term Treasury bonds or the custom LDI portfolio. Even though those portfolios hold more volatile bonds, the risks of those portfolios are less than the risk of the long-only efficient 60/40 portfolio. That is because the long bonds are highly correlated with the liabilities and, as a result, provide a hedge. The portfolios are slightly below the associated efficient frontiers. That is because we simply replaced the fixed income component of the 60/40 portfolio, while the portfolios on the efficient frontiers are optimized.

*Figure 2b* combines the use of LDI with the tangent portfolio concept. The orange line is the same asset-liability efficient frontier that we saw in *Figure 2a*. On it we identify the portfolio that contains the 60% equity component of the efficient long-only 60/40 portfolio and 40% LDI. That portfolio is often described using terminology such as "60% return-seeking and 40% liability hedge."

5 The source for the liability data in Figures 2a, 2b, and 2c are proprietary indexes created by Mellon Pension Services. Please see page the Disclosure Statements page of this publication for more information on these indexes.

6 The active risk of the plan is the risk of a portfolio that is long the assets and short the liabilities. Its variance is equal to the variance of the assets plus the variance of the liabilities, minus the covariance of the assets with the liabilities. As a result, unless the correlation of the assets and liabilities is quite high, the active risk is likely to be higher than the asset-only risk.

7 Since the liabilities are assumed to be fixed, we can ignore their expected return in this analysis.

8 In this paper we assume that the liabilities have a volatility of 11%. That is based on data provided by Mellon Pension Services. The asset-only minimum-risk portfolio is mostly invested in an aggregate bond fund. The active risk of the combination of it and the liabilities is higher than the asset-only risk because aggregate bonds have much shorter duration and, as a result, do not provide a very good hedge for liabilities. Active risk is lower than the risk of the liabilities because aggregate bonds do provide some hedging.

9 The LDI portfolio is designed to precisely match the expected cash flows of the liabilities. Consequently, it is more highly correlated with the liabilities than the "optimal" combination of long government and long corporate bonds, and it provides a better hedge.

The blue line is the efficient frontier that incorporates the use of leverage. It is analogous to the Capital Market Line in the asset-only analysis. The minimum-risk portfolio is the same for both efficient frontiers: 100% LDI. The reason that the blue line provides much better risk-adjusted returns is that through the use of leverage we can maintain the 100% exposure to the LDI portfolio and also hold the most efficient return-seeking portfolio, which is the tangent portfolio from *Figure 1c*. As a result, when we increase the expected return, we take on only asset risk, and not risk from liability mismatch. In addition, as we increase active risk, we do so by holding the portfolio with the maximum return per unit of risk: the tangent portfolio. By way of contrast, when we don't allow leverage (the orange line), seeking higher expected returns adds two sources of risk: reduced exposure to the LDI portfolio and risky positions in other assets.

In addition, even with leverage, if we don't employ the tangent portfolio, we end up with a less efficient portfolio in this illustration. The point labeled "60% return-seeking and 93% LDI" illustrates that fact. It is the portfolio that has the minimum active risk but also includes the 60% equity component from the efficient 60/40 portfolio on the asset-only efficient frontier.<sup>10</sup> It is significantly below the efficient frontier with leverage. That is because the equity portfolio has a lower Sharpe ratio than the tangent portfolio. All of its excess return comes from equities, while the tangent portfolio theoretically consists of the optimal combination of equities and fixed income.

The "Efficient Frontier with Leverage" (blue line) on *Figure 2b* is based on the unrealistic assumption that there aren't any limits on leverage. In fact, leverage is limited by the requirement that one post variation margin. In addition, there are many institutional constraints that limit plans' ability to take on significant leverage. To address the impact of those limits, *Figure 2c* shows that, in our view, even modest amounts of leverage can improve a plan's risk-adjusted return. In addition to the long-only and unconstrained efficient frontiers that are on *Figure 2b*, *Figure 2c* has efficient frontiers that are constrained to maximum leverage of 10% and 25%.<sup>11</sup>

The constrained efficient frontiers follow the unconstrained efficient frontier until they hit the leverage constraint. After that, it appears that they maintain the benefit of adding the high Sharpe ratio tangent portfolio via the use of leverage, but the incremental benefit from leverage becomes constant and begins to show a similar incremental risk-return trade-off to the long-only efficient frontier.

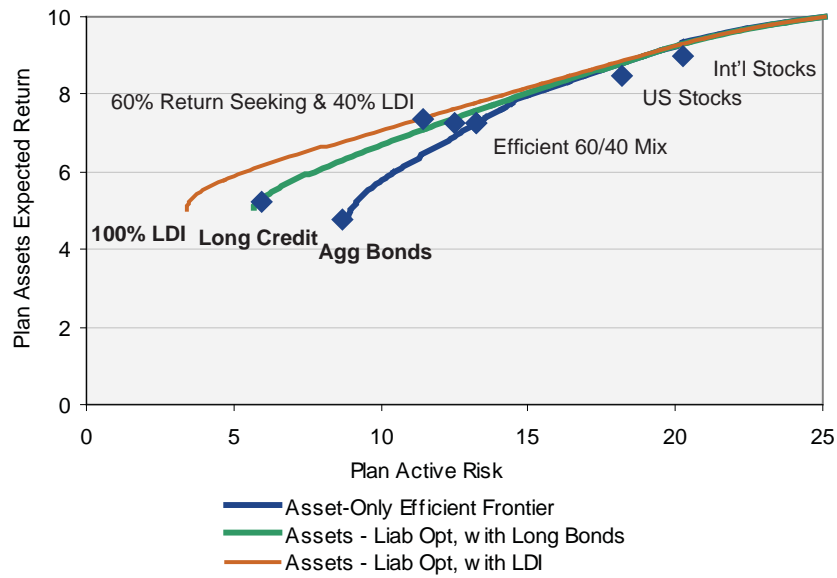
The arrows on *Figure 2c* illustrate the range of outcomes one could achieve starting with the 60/40 portfolio and utilizing 25% leverage. One could keep the same risk and increase the expected return by more than 1%, or one could reduce plan risk by 2.5% and potentially still maintain the expected return.

<sup>10</sup> When we combine the equity positions from the 60/40 efficient portfolio, the minimum risk portfolio contains only 93% LDI. That is because the equity portfolio does provide some liability hedging.

<sup>11</sup> For example, for a plan with \$1 billion in assets and \$1 billion in liabilities, 10% leverage would result in a portfolio consisting of \$1 billion of LDI exposure and \$100 million in exposures to other return-seeking assets.

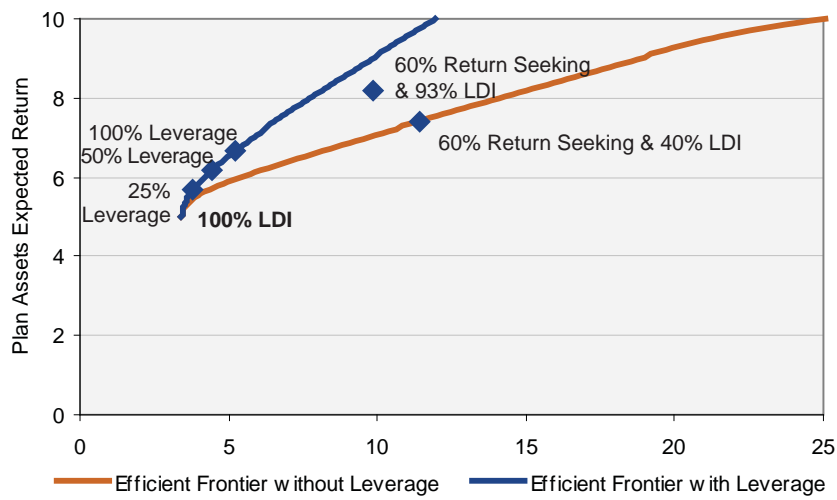
**Figure 2a: Asset Minus Liability Efficient Frontier**

Data Source: Mellon Pension Services



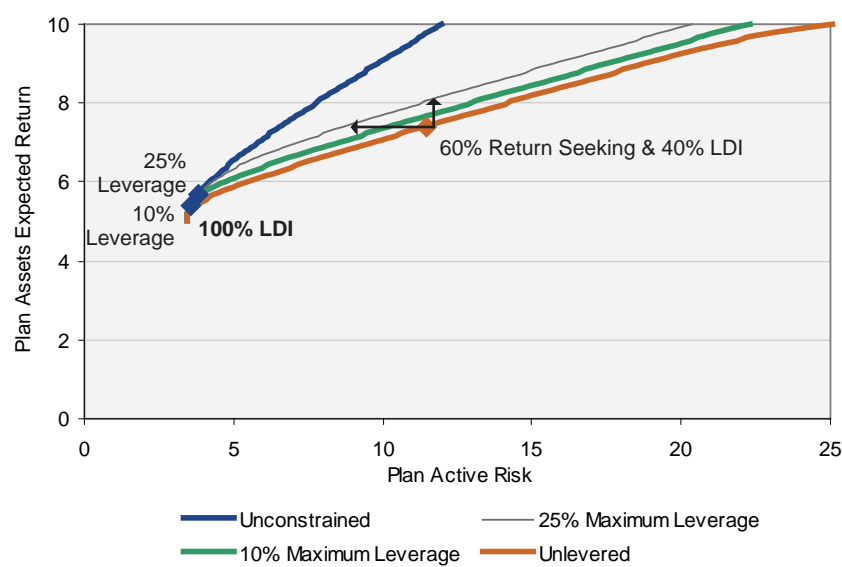
**Figure 2b: Asset Minus Liability Efficient Frontier, Plus Levering the Tangent Portfolio**

Data Source: Mellon Pension Services



**Figure 2c: Asset-Liability Efficient Frontier, Plus Tangent Portfolio, Limits on Leverage**

Data Source: Mellon Pension Services



## CONCLUSION

Sponsors of defined benefit pension plans must balance two conflicting goals. One is to minimize the impact of fluctuation of plan assets on the sponsor. The other is to manage the cost of the plan. Reducing the plan's risk should move one in the direction of adopting an LDI strategy. The traditional way to increase a plan's expected return is to move away from bonds and toward equities. It is common for clients to describe their asset allocation in terminology such as "40% hedged and 60% return-seeking."

The problem with the traditional approach is that moving out the long-only efficient frontier adds two kinds of risk: underweighting the liability hedge and risk from the return-seeking positions. We believe the proper way to describe the 40% hedged and 60% return-seeking portfolio is: 60% return-seeking, and net 60% short the liabilities (100% short liabilities and 40% hedged.)

By combining the tangent portfolio concept with LDI, we believe active risk can be managed more efficiently. The liability hedge is maintained, and the return-seeking portfolio is the tangent portfolio that typically has better risk-adjusted returns and a higher Sharpe ratio than one can achieve by simply moving out the long-only efficient frontier. To put this concept in tangible terms, think about starting with a 100% LDI portfolio and deciding to take on some active risk in order to increase the plan's expected return. The traditional way would be to sell bonds and add equity beta in order to capture the equity risk premium. However, in our view, adding the tangent portfolio via leverage is more efficient. It adds a balanced combination of equity beta and bond term premium, with a goal of providing more expected return for the active risk without sacrificing some of the liability hedge.

## CONTACT US

**BUSINESS DEVELOPMENT** Mike Kotarski, CFA, Managing Director  
617.722.7134  
mikek@mcm.com

**CLIENT SERVICE** Earl Kleckner, Managing Director  
412.236.3936  
earlk@mcm.com

**CONSULTANT RELATIONS** Andy Pellegrino, Managing Director  
412.234.1909  
andyp@mcm.com

**CORPORATE OFFICE** San Francisco  
50 Fremont Street  
Suite 3900  
San Francisco, CA 94105  
415.546.6056

**WEB** [www.mcm.com](http://www.mcm.com)

## DISCLOSURE STATEMENTS

This publication reflects the opinion of the authors as of the date noted and is subject to change without notice. The information in this publication has been developed internally and/or obtained from sources we believe to be reliable; however, Mellon Capital does not guarantee the accuracy or completeness of such information. This publication is provided for informational purposes only and is not provided as a sales or advertising communication nor does it constitute investment advice or a recommendation for any particular investment product or strategy for any particular investor. Economic forecasts and estimated data reflect subjective judgments and assumptions and unexpected events may occur. Therefore, there can be no assurance that developments will transpire as forecasted in this publication. Past performance is not an indication of future performance.

The BNY Mellon Pension Liability Indexes are calculated using the present values of hypothetical Retired, Mature, Typical, and Young benefit liability cash flow schedules, as calculated by BNY Mellon research. These cash flows are discounted according to a proprietary term structure model applied to every forward payment date. Pricing for the term structure model is developed from BNY Mellon internal research. Returns for each of the BNY Mellon Pension Liability Indexes are calculated from monthly changes in the present values of each index. Index valuations and return calculations are performed using two sets of data. The Reporting Basis discounting uses high-grade corporate bond yields to fit a proprietary BNY Mellon term structure model. This method is intended to provide a reasonable approximation of the methodologies generally used for accounting and funding purposes. The Market Value Basis discounting uses U.S. Treasury bond yields to fit a proprietary BNY Mellon term structure model. This method is intended to provide a reasonable approximation of the cost to purchase annuities for the liabilities.